United Kingdom
Mathematics Trust

Junior Mathematical Olympiad

Tuesday 15 June 2021

England & Wales:	*Year 8 or below*
Scotland:	*S2 or below*
Northern Ireland:	*Year 9 or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. Write on one side of the paper only and start each question on a fresh sheet.
5. Write your participant ID and question number neatly in the top left corner of each page and arrange them with your cover and section A answer sheet on top, so that your teacher can easily upload them to the marking platform. **Do not hand in rough work**.
6. Your answers should be fully simplified and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. Only answers are required to the questions in Section A.
8. For questions in Section B, you should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Junior Mathematical Olympiad should be sent to:

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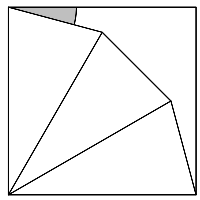
☎ 0113 365 1121 enquiry@ukmt.org.uk www.ukmt.org.uk

Section A

Try to complete Section A within 30 minutes or so. Only answers are required.

A1. What is the value of $\left(1 + \frac{1}{1^2}\right)\left(2 + \frac{1}{2^2}\right)\left(3 + \frac{1}{3^2}\right)$?

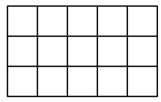
A2. Three identical isosceles triangles fit exactly (without overlap) into a square, with two of the triangles having edges in common with the square, as shown in the diagram.
What is the size, in degrees, of the shaded angle?



A3. What is the value of $\left(\frac{4}{5}\right)^3$ as a decimal?

A4. In the first week after Bill's birthday, his uncle gave him some money for his new piggy bank. Every week after that, Bill put £2 into his piggy bank.
By the end of the ninth week after his birthday, Bill had trebled the amount his uncle gave him. How much, in pounds, did Bill have in total at the end of the ninth week?

A5. Aston wants to colour exactly three of the cells in the grid shown so that the coloured grid has rotational symmetry of order two. Each of the cells in the grid is a square. In how many ways can Aston do this?

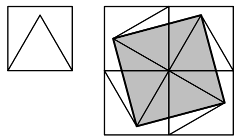


A6. To travel the 140 kilometres between Glasgow and Dundee, Sam travels half an hour by bus and two hours by train. The train travels 20 km/h faster than the bus. The bus and the train both travel at constant speeds. What is the speed, in km/h, of the bus?

A7. The product of five different integers is 12. What is the largest of the integers?

A8. In my desk, the number of pencils and pens was in the ratio 4 : 5. I took out a pen and replaced it with a pencil and now the ratio is 7 : 8. What is the total number of pencils and pens in my desk?

A9. A unit square has an equilateral triangle drawn inside it, with a common edge. Four of these squares and triangles are placed together to make a larger square. Four vertices of the triangles are joined up to form a square, which is shaded and shown in the diagram. What is the area of the shaded square?



A10. Amy, Bruce, Chris, Donna and Eve had a race. When asked in which order they finished, they all answered with a true and a false statement as follows:

Amy: Bruce came second and I finished in third place.

Bruce: I finished second and Eve was fourth.

Chris: I won and Donna came second.

Donna: I was third and Chris came last.

Eve: I came fourth and Amy won.

In which order did the participants finish?

Section B

Your solutions to Section B will have a major effect on your result.

Concentrate firstly on one or two Section B questions and then write out *full solutions* (not just brief 'answers'), including mathematical reasons as to why your method is correct.

You will have done well if you hand in full solutions to two or more Section B questions.

Do *not* hand in rough work.

B1. The solution to each clue of this crossnumber is a two-digit number, that does not begin with a zero.

ACROSS	DOWN
1. A prime	1. A square
3. A square	2. A square

1	2
3	

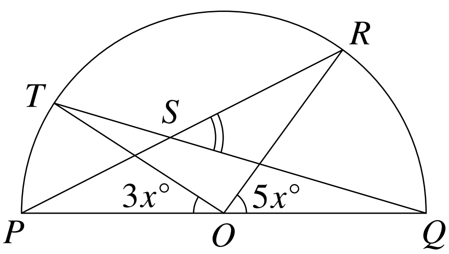
Find all the different ways in which the crossnumber can be completed correctly.

B2. The Smith family went to a restaurant and bought two Pizzas, three Chillies and four Pastas. They paid £53 in total.

The Patel family went to the same restaurant and bought five of the same Pizzas, six of the same Chillies and seven of the same Pastas. They paid £107 in total.

How much more does a Pizza cost than a Pasta?

B3. Two overlapping triangles POR and QOT are such that points P, Q, R and T lie on the arc of a semicircle of centre O and diameter PQ, as shown in the diagram.



Lines QT and PR intersect at the point S. Angle TOP is $3x°$ and angle ROQ is $5x°$.

Show that angle RSQ is $4x°$.

B4. The letters A, B and C stand for different, non-zero digits.

Find all the possible solutions to the word-sum shown.

$$\begin{array}{r} A\ B\ C \\ B\ C\ A \\ +\ C\ A\ B \\ \hline A\ B\ B\ C \end{array}$$

B5. In Sally's sequence, every term after the second is equal to the sum of the previous two terms. Also, every term is a positive integer. Her eighth term is 400.

Find the minimum value of the third term in Sally's sequence.

B6. The integers 1 to 4 are positioned in a 6 by 6 square grid as shown and cannot be moved.

	1			2	
	3			4	

The integers 5 to 36 are now placed in the 32 empty squares. Prove that no matter how this is done, the integers in some pair of adjacent squares (i.e. squares sharing an edge) must differ by at least 16.

